Our calibrated model has no predictive value: An example from the petroleum industry

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In many studies involving numeric models of complex real world situations, for example petroleum reservoirs and climate modelling, it is implicitly assumed that if the model has been carefully calibrated to reproduce previously observed behaviour, then the model will have some predictive capacity. It is recognised that predictability may only be achievable for a finite period of time, and that any prediction will be uncertain to some extent.

Two types of error are considered in most calibration exercises: measurement error and model error. Measurement errors are fixed at the time the measurement was made, they generally have well defined statistics and can be handled appropriately. Model errors are due to approximations, such as a loss of spatial, or temporal, resolution, and the non-inclusion of all of the relevant physics. The assumption that is normally made is that if the model errors are sufficiently unimportant, so that when the model has been calibrated to measurement data, then we have some level of acceptable predictability. If the model does not have predictability, then the model errors are assumed to be too large and we need to use a "better" model. Where "better" probably means improved resolution, spatial or temporal, and/or the inclusion of more physics.

In this paper we present the results of a study, for a petroleum reservoir, that suggests that:

- In the absence of model errors, and with very low measurement errors, it is possible to obtain calibrated models that do not have any predictive capability.
- That such models may be significantly easier to identify than the correct model.
- We are unable to differentiate between calibrated models with or without predictive capabilities.
- The introduction of even small model errors may make it impossible to obtain a calibrated model with predictive value.

If the observations made with this model are not unique to the model, and we have no reason to believe that the model is unique, then this presents a potentially serious obstacle to the use of models of this type for prediction.

Our model is a cross-section of a simple layered reservoir, with a single vertical fault midway between an injector producer pair, as shown in figure 1. The model that we

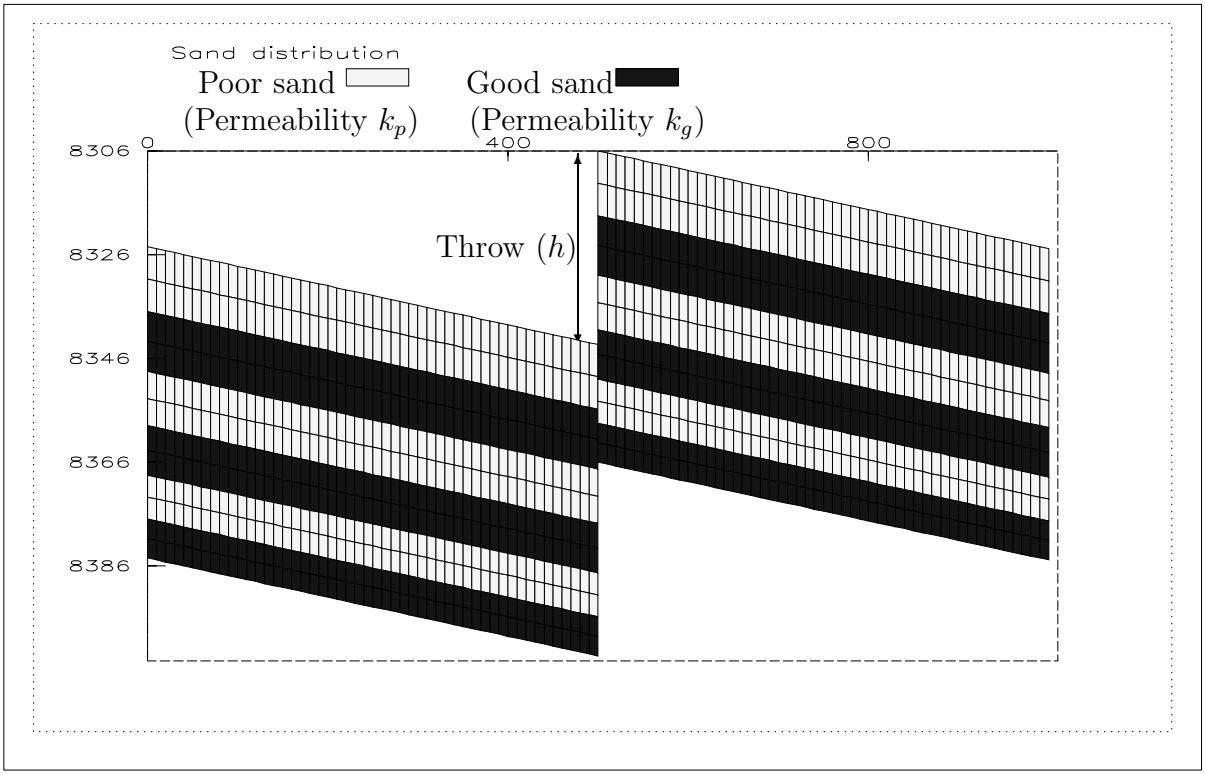

Figure 1: Reservoir model

calibrate has three parameters: the vertical displacement (throw) of the fault; the permeability of the poor quality sand; and the permeability of the good quality sand. The geological layers are assumed to be homogeneous (ie they have constant physical properties). The "truth" case, which is used to generate the measurements for the calibration, is a variant of the calibration model, but with fixed parameter values. In the case of no model error, then the "truth" case is a member of the set of all possible calibration models. The size and type of model error is chosen by how a specific calibration model is perturbed to obtain the truth case. In the work presented in this paper, the model error is obtained by introducing small variations into the spatial properties of the geological layers. The permeability and porosity in each grid block are randomly perturbed. The maximum variations that are allowed is $\pm 1\%$ of the unperturbed mean values. These perturbations are much lower than would be expected for a real world rock that had been classified as homogeneous.

Our procedure is as follows:

1. Choose "truth" values for the three model parameters;
2. Select the level of measurement and model error to be used;
3. From the truth case produce the measurements required for the calibration process (three years of monthly data);
4. Calibrate the model against the measurements;
5. Predict the behaviour for years 4-10.

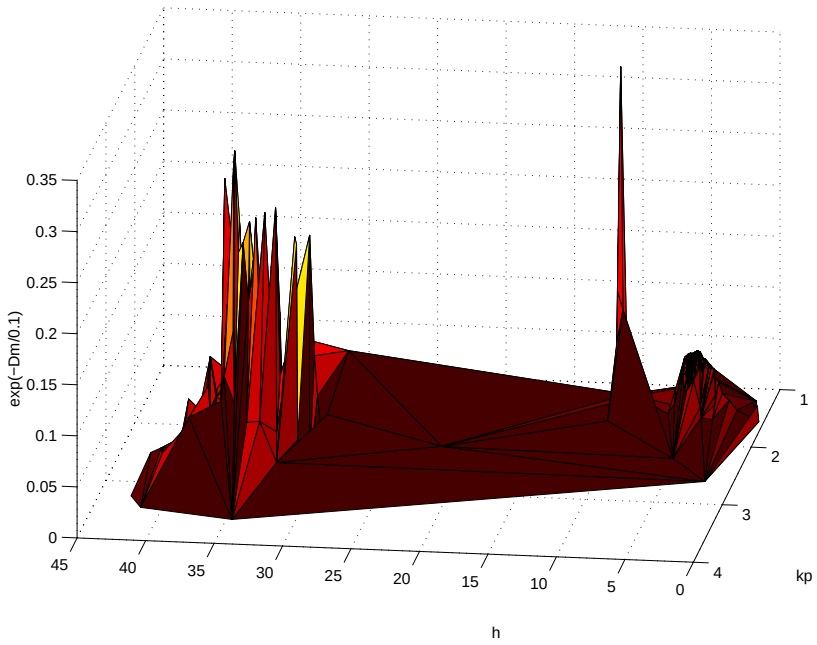

Figure 2: Evolved population of models. Neighbouring models are connected by triangles.

We have considered the truth case: $h = 10.4$, $k_p = 1.31$ and $k_g = 131.7$ with and without model error. No measurement error was added, but we assumed Gaussian noise with a 1% standard deviation when calculating the likelihood that a proposed calibration matches the truth.

In order to quantify the degree of the model calibration against measurements, we define first an objective function for the calibration period, Δ_m, as follows

$$\triangle_m = \frac{1}{36} \sum_{j=1}^{36} \sum_{k=1}^{3} \frac{|sim(j,k) - obj(j,k)|}{2\sigma_{jk}} \tag{1}$$

where $sim(j,k)$ is the simulated response for production serie k of the model at time t, $obj(j,k)$ is the corresponding true value and σ_{jk}, an estimation of what would be the associated measurement error. We consider three production series: Oil Production Rate, Water Production Rate (or Water Cut) and Water Injection Rate.

Likewise, the objective function for the prediction period, Δ_f, is

$$\triangle_f = \frac{1}{7} \sum_{j=37}^{43} \sum_{k=1}^{3} \frac{|sim(j,k) - obj(j,k)|}{2\sigma_{jk}} \tag{2}$$

Thereafter, we run a Steady State Real-Parameter Genetic Algorithm, using Δ_m as the fitness measure, for the truth without model error. This GA used a population size of 300 models and a total of $12,300$ forward simulations. A representation of the evolved population of models is presented in figure 2.

As pictured in figure 2, the GA has revealed three optimal regions. It is to be noted that the basin surrounding the global optimum is very small compared to the basin surrounding

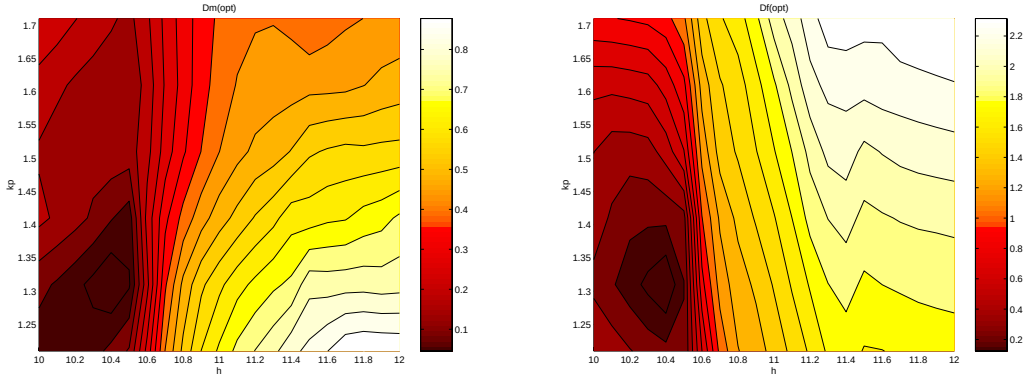

Figure 3: Surface plots for $\Delta_m(\text{opt})$, and $\Delta_f(\text{opt})$ in region 1 without model error, $h \in (10, 12)$ and $k_p \in (1.20, 1.70)$

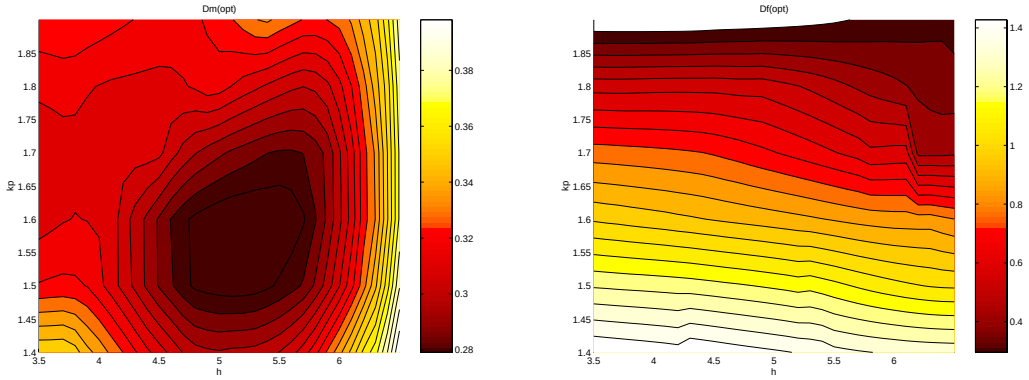

Figure 4: Surface plots for $\Delta_m(\text{opt})$, and $\Delta_f(\text{opt})$ in region 2 without model error, $h \in (3.5, 6.5)$ and $k_p \in (1.40, 1.90)$

the other significant optimum. Thus, most optimisation algorithms are likely to find a non-global optimum, whilst not finding the global optimum.

We now perform an exhaustive search on a regular grid for each of these three interesting regions. The grid resolution is $\triangle_h = 0.1$, $\triangle_{k_p} = 0.1$ and $\triangle_{k_g} = 0.01$. We know from numerous numerical studies that for fixed values of (h, k_p), then there is a unique minimum, as a function of k_g, in our objective function. We are therefore able to present our results as follows: for any pair (h, k_p) we can obtain $k_g(\text{opt})$, $\Delta_m(\text{opt})$, and $\Delta_f(\text{opt})$; hence we can produce surface maps of these quantities for a range of (h, k_p). Figures 3, 4 and 5 show the results for the case in which there is no modelling error. We can see that only the global optimum makes an accurate prediction, this is further illustrated by the plots of water injection, oil and water production rates in figure 6.

When we repeat these calculations, but having introduced the small model error to obtain figures 7, 8 and 9. We can see that there is no longer a optimum at about $(h, k_p) = (10.4, 1.31)$. The global optimum has clearly moved away from the "truth". Neither does the "truth" represent a local optimum for the prediction. The local optimum in region 2 has also disappeared. I region 3 the local optimum has moved, but there is no

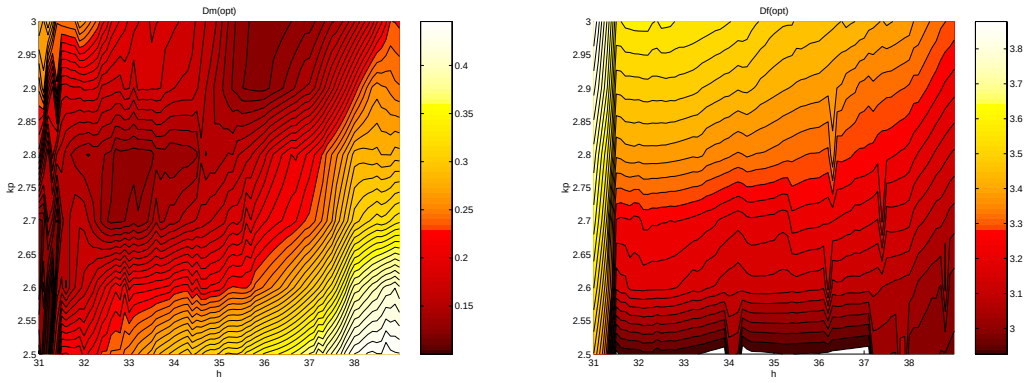

Figure 5: Surface plots for $\Delta_m(\text{opt})$, and $\Delta_f(\text{opt})$ in region 3 without model error, $h \in (31, 39)$ and $k_p \in (2.50, 3.00)$

associated local optimum for the prediction.

Qualitively there is no difference between the processes used in the two cases. However the results that we obtain are quite different. Given our inability, for this model, to know the value of the prediction from the available information, it is difficult to be confident that if we have a well calibrated model that its predictions have value or not.

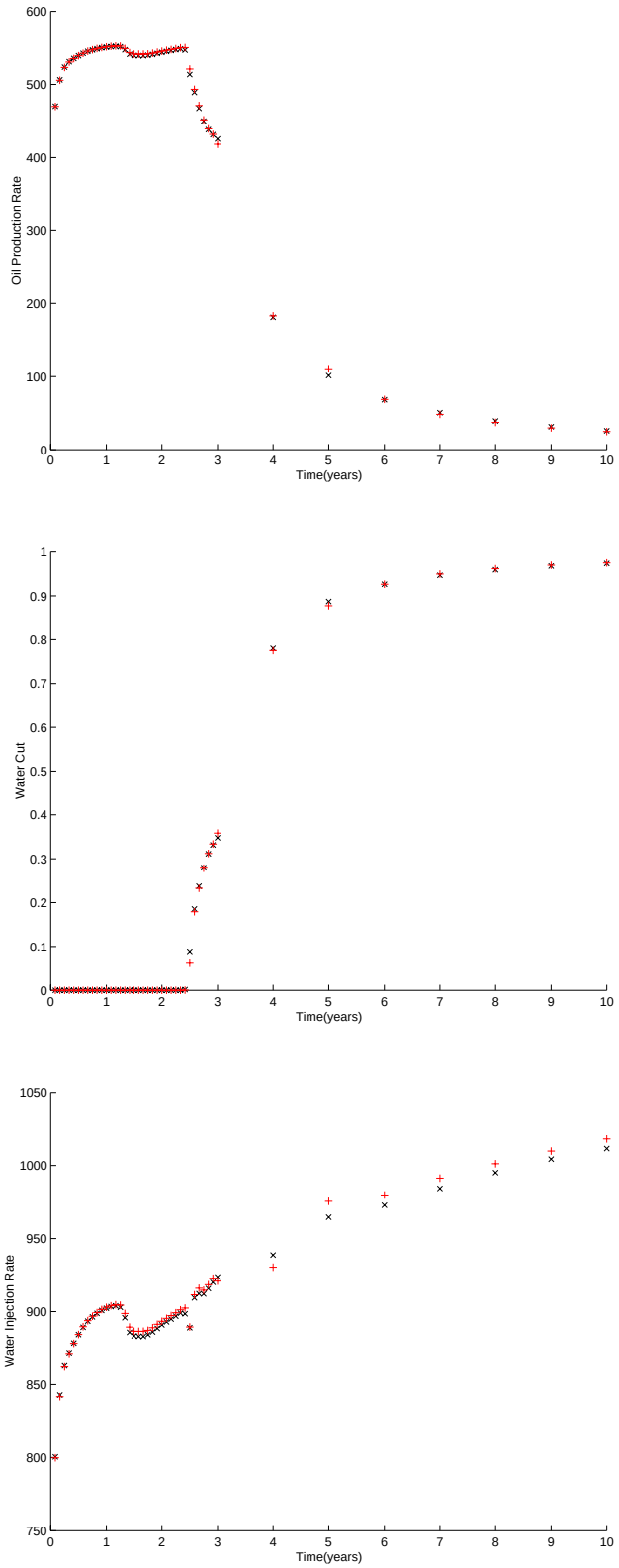

Figure 6: Calibration (up to 3 years) and prediction (3 years onwards) for the best model ($h = 10.5$, $k_p = 1.53$ and $k_g = 131.3$) provided by the GA. $\Delta_m = 0.11$ and $\Delta_f = 0.85$. "$\times$" are the true measurements and "+" the simulated values.

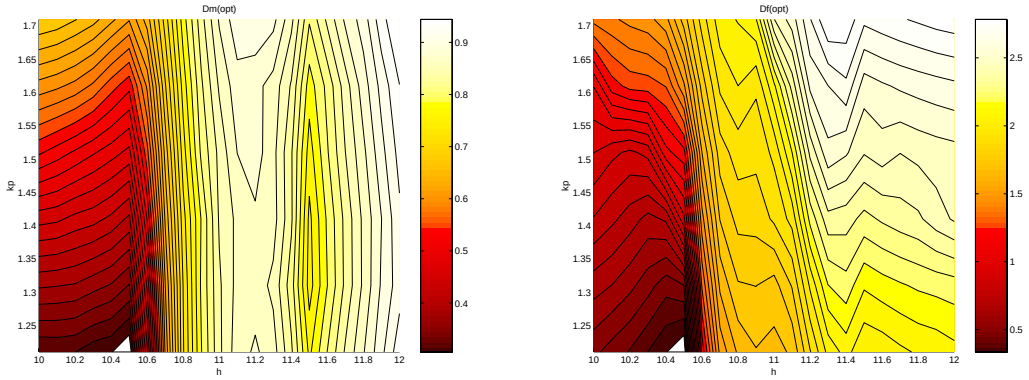

Figure 7: Surface plots for $\Delta_m(\text{opt})$, and $\Delta_f(\text{opt})$ in region 1 with model error, $h \in (10, 12)$ and $k_p \in (1.20, 1.70)$

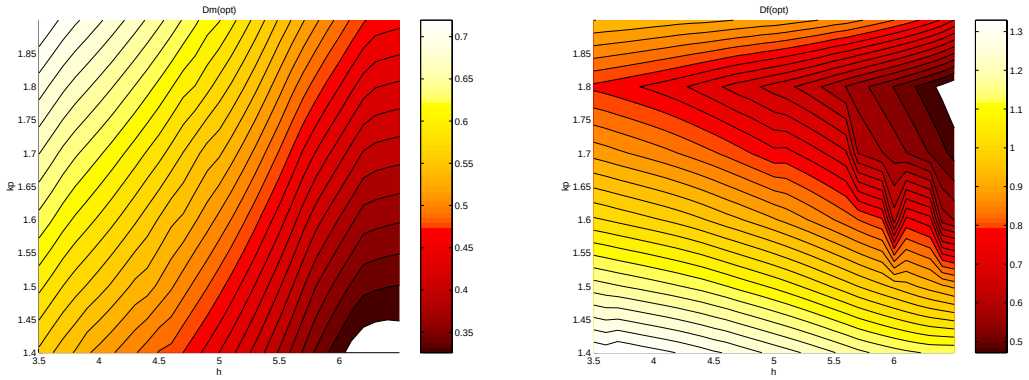

Figure 8: Surface plots for $\Delta_m(\text{opt})$, and $\Delta_f(\text{opt})$ in region 2 with model error, $h \in (3.5, 6.5)$ and $k_p \in (1.40, 1.90)$

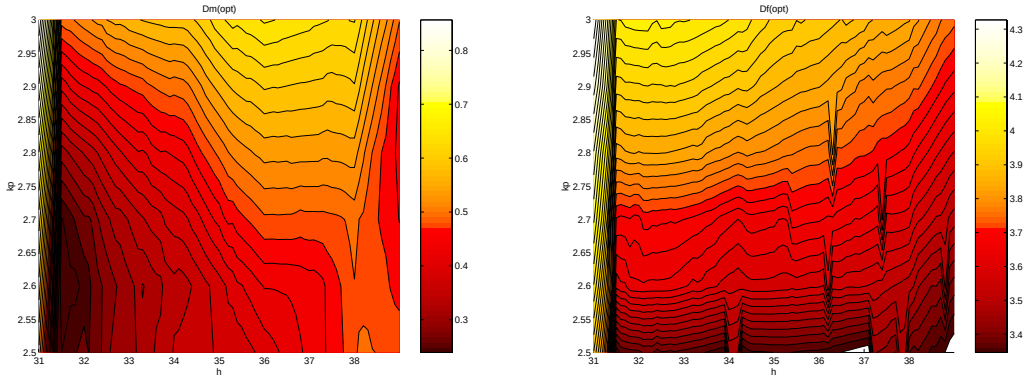

Figure 9: Surface plots for $\Delta_m(\text{opt})$, and $\Delta_f(\text{opt})$ in region 3 with model error, $h \in (31, 39)$ and $k_p \in (2.50, 3.00)$